What is the probability of 3 people having the same birthday in Derry Lab? (Short Version)

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Calculate the Probability such that no more than 2 people share a birthday

Case 1: No one shares a birthday (i.e. everyone has a different birthday)

$$p_0 = \frac{365}{365} \times \frac{364}{365} \times \cdots \times \frac{365 - 14 + 1}{365}$$

$$p_0 = \frac{365P_{14}}{365^{14}}$$

Case 2: There are pairs (2 people) having the same birthday

1 pair shares a birthday.

$$p_1 = \frac{14C_2}{1!} \times \frac{365P_{13}}{365^{13}}$$

2 pairs each share their birthdays.

$$p_2 = \frac{14C_2 \times 12C_2}{2!} \times \frac{365P_{12}}{365^{14}}$$

Summation of all cases:

$$p_{sum} = \sum_{k=1}^{7} \left(\frac{\prod_{n=0}^{k-1} 14-2nC_2}{k!} \times \frac{365P_{14-k}}{365^{14}} \right)$$

The probability of at least 3 people sharing a birthday in a lab of 14 people is

$$P = 1 - (p_0 + p_{sum})$$
$$P = 0.0026695...$$
$$P \approx 0.27\%$$